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BERTRAND COMPETITION

This article presents the classic Bertrand model of oligopolistic price competition and shows how alternative assumptions on economic primitives – such as the structure of demand and cost functions, tie-breaking rules, and product differentiation – shape Nash equilibrium prices and profits. We also discuss the related Bertrand–Edgeworth model of price competition in which consumers may be rationed – either strategically or due to capacity constraints – and illustrate how alternative rationing rules influence equilibrium.

‘Bertrand competition’ refers to a model of oligopoly in which two or more firms compete by simultaneously setting prices and in which each firm is committed to provide consumers with the quantity of the firm’s product they demand given these ‘posted prices’. The concept is named after the French mathematician Joseph Louis François Bertrand (1822–1900) who, in an 1883 review of Cournot (1838), was critical of Cournot’s use of quantity as the strategic variable in his famous duopoly model of market rivalry. In his critique, Bertrand described how, in Cournot’s duopoly environment where identical firms produce a homogeneous product under a constant unit cost technology, price competition would lead to price undercutting and a downward spiral of prices. Bertrand erroneously reasoned that this process would continue indefinitely, thereby precluding the existence of an equilibrium. It is now widely recognized that an equilibrium exists not only in Bertrand’s original formulation but in a plethora of other environments in which firms sell either homogeneous or differentiated products.

Formally, *Bertrand competition* is a normal form game in which each of $n \geq 2$ players (firms), $i = 1, 2, \ldots, n$, simultaneously sets a price $p_i \in P_i = [0, \infty)$. Under the assumption of profit maximization, the payoff to each firm i is $\pi_i(p_i, p_{-i}) = p_i D_i(p_i, p_{-i}) - C_i(D_i(p_i, p_{-i}))$, where p_{-i} denotes the vector of prices charged by all firms other than i, $D_i(p_i, p_{-i})$ represents the total demand for firm i’s product at prices (p_i, p_{-i}), and $C_i(D_i(p_i, p_{-i}))$ is firm i’s total cost of producing the output $D_i(p_i, p_{-i})$. A *Bertrand equilibrium* is a Nash equilibrium of this game; that is, a vector of prices $\left(p_i^*, p_{-i}^*\right)$ such that, for each player i, $\pi_i\left(p_i^*, p_{-i}^*\right) \geq \pi_i\left(p_i, p_{-i}^*\right)$ for all $p_i \in P_i$.

The Bertrand paradox

In the 'classic' model of Bertrand competition, each of the n firms produces an identical product at a constant unit cost of c; that is, $C_i\left(q_i\right) = cq_i$. Since their products are perfect substitutes, firms effectively compete for the total demand, $D\left(p\right)$, that a monopolist serving the entire market would obtain by pricing at $p.$ The firm setting the lowest price gets all of this demand; in the event of a tie, the firms charging the lowest price share total demand equally. Total demand is sufficiently well-behaved to ensure that the corresponding monopoly profit function, $\pi\left(p\right) \equiv pD\left(p\right) - C\left(D\left(p\right)\right)$, is not only continuous, but (*a*) has a unique maximizer, the monopoly price p^M; (*b*) satisfies $\pi\left(p\right) < \pi\left(c\right) = 0$ for $p < c$; and satisfies (*c*) $0 < \pi\left(p\right) < \pi\left(p^M\right) < \infty$ for all $p^M > p > c.$ Despite the continuity of $\pi\left(p\right)$, each firm faces a discontinuous profit function

$$\pi_i\left(p_i, p_{-i}\right) = \begin{cases} \left(p_i - c\right)D\left(p_i\right) & \text{if} \quad p_i < p_j \text{ for all } j \neq i \\ \left(p_i - c\right)D\left(p_i\right)/m & \text{if} \quad i \text{ ties } m-1 \text{ other firms for low price} \\ 0 & \text{otherwise} \end{cases}$$

because a firm that prices even slightly above the lowest price gets no demand. In this classic setting with 'well-behaved' demand and constant marginal cost, $\left(p_i^*, p_{-i}^*\right)$ is a *Bertrand equilibrium* if and only if $p_j^* \geq c$ for every firm j and at least two firms set price equal to c. Consequently, all firms earn zero profits in equilibrium, a result that has come to be known as the *Bertrand paradox*. The paradox stems from the fact that, while a monopolist would earn strictly positive profits by charging a price in excess of marginal cost, it takes only two firms to completely dissipate the monopoly profits and achieve the competitive outcome. In a Bertrand equilibrium, all transactions take place at marginal cost (c), and all firms earn zero profits.

The proof of this proposition follows in part from the original intuition of Bertrand. Since the products are perfect substitutes, consumers will purchase only from a firm that charges the lowest price in the market, $p_L \equiv \min_j p_j$. First, $p_L \leq p^M$

in any equilibrium; otherwise, any firm could profitably deviate by lowering its price to p^M. Second, $p_L \geq c$ in any equilibrium; otherwise, a firm charging p_L (and thus earning strictly negative profits) could profitably deviate by increasing its price to c. Third, if $p^M \geq p_L > c$, then at least one firm could increase its profit by unilaterally undercutting p_L by a small amount. Hence, $p_L = c$ in any equilibrium. Fourth, if only a single firm charged a price of $p_L = c,$ it would earn a payoff of zero, and could increase its price to $p' > c$ (but below the second-lowest price) to earn a positive profit. Thus, in any equilibrium at least two firms charge a price of $p_L = c$. Finally, since the only firms attracting any consumers are those pricing at $p_L = c$, all firms earn zero profits. Furthermore, no firm can unilaterally change its price to earn positive profits.

One consequence of this argument is that when $n = 2$ there is a unique Bertrand equilibrium in the classic model: both firms set the common price $p_1^* = p_2^* = c$. When $n > 2,$ there is a unique symmetric equilibrium (in which $p_i^* = c$ for all i) and a continuum of asymmetric equilibria (where two or more firms price at c and one or more firms charge prices arbitrarily higher than c).

Although the Bertrand paradox result summarized above for the case of identical constant unit costs is stated in terms of pure strategies and a symmetric tie-breaking rule, the paradox also obtains for the extension of strategy spaces to allow for mixed-strategies as well as other tie-breaking rules. Alternative tie-breaking rules include ‘winner-take-all sharing’ (where a fair randomizing device is used to determine the identity of the firm that services the entire market in the event of a tie for the lowest price) and ‘unequal sharing’ (where firms tying for the lowest price receive an unequal fraction of total market demand in the event of a tie for the lowest price).

Baye and Morgan (1999) have shown that if the monopoly profit function, $\pi(p),$ is unbounded, there exists (in addition to the Bertrand paradox equilibria) a continuum of non-degenerate mixed strategy equilibria in which each firm earns positive profits. For instance, suppose market demand is given by $D(p) = p^{\alpha}$, where $\alpha \in (-\infty, -1/n)$ is the elasticity of market demand. In this case, one can show that there is a unique symmetric Cournot (quantity-setting) equilibrium in which each firm earns positive profits and the equilibrium market price is $p^* = \left[n\alpha/(1+n\alpha)\right]c$. In contrast, under Bertrand competition any symmetric profit level $\pi^* \in (0,\infty)$ (including profit levels above the Cournot profit) can be achieved in an (atomless) symmetric mixed strategy equilibrium. Equilibrium mixed strategies that support these positive profit levels are described by the cumulative distribution function $F(p) = 1 - \pi^*/\pi(p)$ on $\left[\pi^{-1}(\pi^*),\infty\right),$ where $\pi(p) = (p-c)p^{\alpha}.$

Even with a bounded monopoly profit function $\pi(p),$ the coexistence of positive profit equilibria and (zero profit) Bertrand paradox equilibria can arise for alternative cost functions and sharing rules. For instance, with a symmetric tie-breaking rule (see Dastidar, 1995), if firms have identical cost functions that are increasing and strictly convex in output, a symmetric zero profit equilibrium may exist in which each firm prices at $p^0,$ where p^0 satisfies $p^0 D(p^0)/n - C(D(p^0)/n) = 0.$ In addition, however, a continuum of positive profit symmetric pure-strategy equilibria can arise in which each firm charges a price contained in an interval above p^0. Intuitively, with strictly convex costs, a firm that deviates by undercutting such a price would increase its demand (and revenues) by a factor of n, but the firm's cost would increase by a factor greater than n.

This result for bounded demand and identical convex costs is based on a symmetric tie-breaking rule; with convex costs, different results generally obtain for other tie-breaking rules. For instance, under the winner-take-all tie-breaking rule (see Baye and Morgan, 2002), any firm charging the price p_L earns a payoff of $\pi(p_L)/\#L,$ where $\#L$ is the number of firms charging the price $p_L.$ In this case, if $\pi(p_L) > 0$, some firm could gain by undercutting p_L by a small amount (a firm pricing above p_L could increase its payoff from zero to $\pi(p_L - \varepsilon) > 0;$ a firm that tied another firm at p_L could increase its profits from $\pi(p_L)/\#L$ to $\pi(p_L - \varepsilon)$ by slightly undercutting p_L). Consequently, an argument similar to that for the case of constant unit costs implies that, with bounded demand and convex costs, any equilibrium under the winner-take-all sharing rule involves at least two firms charging a price p_L such that $\pi(p_L) = 0$, so that the (zero profit) Bertrand paradox is the only configuration of firm profits.

With bounded demand and identical concave costs, a similar argument reveals that any equilibrium under the winner-take-all sharing rule involves at least two firms charging a price p_L such that $\pi(p_L)=0$ (Baye and Morgan, 2002). However, under a symmetric sharing rule, concave costs (increasing returns) are problematic for the existence of a Bertrand equilibrium in either pure or mixed strategies. To illustrate, consider a duopoly in which market demand is given by $D(p)=1-p$ for $p\in[0,1]$, and in which each firm has an identical concave cost function

$$C_i(q_i)=\begin{cases} 0 & \text{if} \quad q_i=0 \\ f+cq_i & \text{if} \quad q_i>0 \end{cases}$$

where $1>c>0$ and $f<[(1-c)/2]^2$. Note that c represents marginal cost and f is a fixed cost that may be avoided by producing zero output. One may readily verify that a monopolist would earn strictly positive profits by pricing at the monopoly price $p^M=(1+c)/2$, and that the minimum 'breakeven price' is $p^0=\left[(1+c)-\left[(1-c)^2-4f\right]^{1/2}\right]/2$; that is, $0=\pi(p^0)>\pi(p)$ for all $p<p^0$. Under a winner-take-all sharing rule, $p_1=p_2=p^0$ is a pure-strategy Nash equilibrium and firms earn zero profits in this 'Bertrand paradox' equilibrium. In contrast, under a symmetric tie-breaking rule there does not exist an equilibrium (in pure or mixed strategies).

The intuition for the failure of existence of equilibrium with a symmetric tie-breaking rule in this example is as follows. Clearly, neither firm has an incentive to price below p^0 (since monopoly profits are negative for such prices and a firm can guarantee a payoff of zero by pricing at $p_i=1$). If both firms priced at p^0 with probability one, symmetric sharing implies that they would earn negative profits, since $C_i\left(D(p^0)/2\right)>C_i\left(D(p^0)\right)/2$. Thus, p^0 is strictly less than the upper bound of the support of at least one firm's (possibly degenerate) mixed strategy. Let $p^H>p^0$ denote highest of the upper bounds of the supports of the two firms' mixed strategies. In any equilibrium, at most one firm has a mass point at p^H; otherwise, there would be a positive probability of a tie at this price and a firm could gain by reallocating mass to lower prices. If there is a mass point at p^H, the firm charging p^H with positive probability must earn its equilibrium profits at this price, which are necessarily zero since it is undercut with certainty. If there is no mass point at p^H, then a firm whose support includes p^H must achieve its equilibrium payoff when pricing at p^H, and since p^H is undercut with certainty, this equilibrium payoff is zero. Therefore, at least one firm i whose support includes p^H earns an equilibrium payoff of zero. Moreover, since firm i earns an equilibrium payoff of zero, p^0 must be the upper bound of the support of the other firm j's mixed strategy; if the upper bound of j's support was $p' \in \left(p^0, p^H\right],$ firm i could increase its profits by reallocating probability mass to some price below p'. Thus, if there is an equilibrium, at least one firm must charge a price of p^0 with probability one. However, since firm i charges prices in the interval $\left[p^0, p^H\right],$ and not all mass is at $p^0,$ it follows that there exists some price $p'' \in \left(p^0, p^H\right]$ such that firm j could gain by reallocating mass from p^0 to p'', a contradiction. Hence, there does not exist an equilibrium in pure or mixed strategies.

Bertrand–Edgeworth competition

In an early critique of Bertrand and Cournot, Edgeworth (1925) observed that the Bertrand paradox may not obtain if firms are capacity constrained. Indeed, in the analysis above, if firm i's demand $D_i\left(p_i, p_{-i}\right)$ is greater than firm i's largest competitive supply at $p_i,$ $s_i\left(p_i\right) \equiv \max\left\{\text{argmax}_q\, p_i q - C_i\left(q\right)\right\},$ then firm i would earn higher profits by supplying a quantity strictly less than that demand and rationing customers. A variant of Bertrand competition, known as 'Bertrand–Edgeworth competition', allows any firm to ration the demand that it faces at given prices by only providing its optimal or competitive supply at its price. Rationing may stem from a physical capacity constraint, $k_i,$ that prevents firm i from producing more than k_i units (as in Edgeworth's original formulation), or more generally, from a firm's strategic incentive to refuse to fulfill the quantity demanded of all consumers at a given price. Under Bertrand–Edgeworth competition one must therefore specify how demand is rationed when a firm's quantity demanded at given prices exceeds the amount of product it produces.

Two prominent rationing rules used in this context are efficient rationing (in which case the good is first allocated to consumers who most highly value the product) and proportional rationing (in which case the good is allocated to a fraction of consumers without regard to their valuations of the product). In the duopoly case, for instance, efficient rationing means that if $p_i > p_j$, firm i's 'residual' demand is $D_i\left(p_1, p_2\right) \equiv \max\left\{0, D\left(p_i\right) - s_j\left(p_j\right)\right\}$. Under proportional rationing, firm i's demand is $D_i\left(p_1, p_2\right) \equiv \max\left\{0, D\left(p_i\right)\left[1 - s_j\left(p_j\right)/D\left(p_j\right)\right]\right\}$. Under both rationing rules, the firm charging the lowest price enjoys a demand of $D\left(p_j\right)$. It is typically assumed that, in the event of a tie, total demand is allocated in proportion to firms' competitive supplies; that is, if both firms charge a price of p, firm i gets a share $\alpha_i \equiv s_i\left(p\right)/\left(s_1\left(p\right) + s_2\left(p\right)\right)$.

For the special case of a duopoly in which each firm has a constant marginal cost (c) up to a capacity of k_i, the cost functions are:

$$C_i\left(q_i\right) = \begin{cases} cq_i & \text{if} \quad 0 \le q_i \le k_i \\ \infty & \text{if} \quad\quad q_i > k_i \end{cases}$$

In this case, under the assumption of well-behaved demand, $s_i\left(p_i\right) = k_i$ for all $p_i \ge c$; that is, each firm opts for a 'corner solution' at full capacity when price exceeds marginal cost. Under both efficient and proportional rationing, if $D\left(c\right) \le k_i,\ i = 1, 2$, then neither firm's capacity constraint ever binds and the Bertrand paradox arises under the same conditions as set forth above; the unique equilibrium is $p_1^* = p_2^* = c$. Characterization of equilibrium when one or more firms is capacity constrained at a price equal to c depends on whether each firm is capacity constrained at its 'residual monopoly price' when its rival sets $p_j = D^{-1}\left(k_1 + k_2\right)$. The term 'residual monopoly price' refers to a firm's optimal price, given its capacity constraint and residual demand (the demand that remains after the other firm has sold its capacity). Note that, in equilibrium, neither firm would ever set a price below $D^{-1}\left(k_1 + k_2\right)$, for at such a price total demand exceeds total capacity, and a firm could increase its price without losing sales. Characterization of equilibrium when $D\left(c\right) > k_i$ for one or more firms then depends on whether $p_1 = p_2 = D^{-1}\left(k_1 + k_2\right)$ is an equilibrium. If, for each firm

i, $D^{-1}(k_1+k_2)$ is the residual monopoly price when firm j sets $p_j = D^{-1}(k_1+k_2)$, then $p_1^* = p_2^* = D^{-1}(k_1+k_2)$ is the unique Bertrand–Edgeworth equilibrium. If some firm i's residual monopoly price exceeds $D^{-1}(k_1+k_2)$ when $p_j = D^{-1}(k_1+k_2)$, then the unique equilibrium is in non-degenerate mixed-strategies.

The residual monopoly price depends on the rationing rule. For proportional rationing, $D_i(p_1,p_2) \equiv \max\left\{0, D(p_i)\left[1-k_j/D(p_j)\right]\right\}$ for any given p_j, and hence firm i's demand is proportional to $D(p_i)$. This implies that, ignoring firm i's capacity constraint, the residual monopoly price based on $D_i(p_1,p_2)$ corresponds to the standard monopoly price, $p^M \equiv \operatorname{argmax}_p\left\{(p-c)D(p)\right\}$. When $p_j = D^{-1}(k_1+k_2) < p^M$, firm i has sufficient capacity to satisfy residual demand at p^M, and hence p^M is firm i's residual monopoly price; if $p_j = D^{-1}(k_1+k_2) \geq p^M$, concavity of the monopoly profit function implies that $p_i = D^{-1}(k_1+k_2)$ is firm i's residual monopoly price. It follows that, for proportional rationing, $p_1^* = p_2^* = D^{-1}(k_1+k_2)$ is the unique Bertrand–Edgeworth equilibrium as long as $D^{-1}(k_1+k_2) \geq p^M$.

Under efficient rationing, $D_i(p_1,p_2) \equiv \max\left\{0, D(p_i) - k_j\right\}$, so that ignoring firm i's capacity constraint, the residual monopoly price is $p_i^R = \operatorname{argmax}_{p_i}\left\{(p_i - c)\max\left(0, D(p_i) - k_j\right)\right\}$. It follows that $p_i^R < p^M$. When $p_j = D^{-1}(k_1+k_2) < p_i^R$, firm i has sufficient capacity to satisfy residual demand at p_i^R, and hence p_i^R is firm i's residual monopoly price; if $p_j = D^{-1}(k_1+k_2) \geq p_i^R$, concavity of the monopoly profit function implies that $p_i = D^{-1}(k_1+k_2)$ is firm i's residual monopoly price. Hence, $D^{-1}(k_1+k_2)$ is firm i's residual monopoly price when firm j sets $p_j = D^{-1}(k_1+k_2)$ if and only if $D^{-1}(k_1+k_2) \geq p_i^R$. This implies that the region in which a pure strategy equilibrium arises is larger for the case of efficient rationing than under proportional rationing. In fact, since the unconstrained residual profit-maximization problem faced by firm i under efficient rationing may be written in terms of either price or quantity, p_i^R is the price arising in a Cournot setting where firm i's output is a best response to an output of k_j by the rival. Hence, if k_i is less than or equal to firm i's Cournot best response to k_j, firm i is capacity constrained and its residual monopoly price equals $D^{-1}(k_1+k_2)$. Consequently, $p_1^* = p_2^* = D^{-1}(k_1+k_2)$ is the unique Bertrand–Edgeworth equilibrium when each firm's capacity is less than or equal to its Cournot best response (given unit cost c) to the other firm's capacity.

Outside of the above regions of capacity, the only Bertrand–Edgeworth equilibria are in non-degenerate mixed strategies in which firms randomize prices over a common interval of prices that exceed c and earn positive expected profits. This corresponds to the regions of capacities in which 'Edgeworth cycles' arise (Edgeworth, 1925). As before, these mixed strategies depend on the rationing rule. For proportional rationing, these mixed strategies are generally difficult to derive; see Davidson and Deneckere (1986) for a characterization. For efficient rationing, these mixed strategies have been characterized by Kreps and Scheinkman (1983), and entail the firm with the larger capacity earning an expected payoff that equals the monopoly profit associated with the residual demand (with symmetric capacities, each firm earns this expected payoff). The firm with the larger capacity earns the higher payoff.

To summarize, only two types of pure-strategy equilibria exist under Bertrand–Edgeworth duopoly with constant unit cost. When capacity constraints do not bind, the classic Bertrand equilibrium arises and the unique equilibrium is for each firm to price at marginal cost to earn zero profits. When capacities are sufficiently small, firms price above marginal cost (at a price that clears all capacity) and earn positive profits in the unique Bertrand–Edgeworth equilibrium. When capacities are in an intermediate range, the equilibrium is generally unique, but in non-degenerate mixed strategies. Firms' prices exceed marginal cost with probability one, and firms earn positive profits.

Positive profit equilibria can also arise in homogeneous product Bertrand settings in which firms endogenously choose capacities. Specifically, consider a two-stage game where, in the first stage, firms simultaneously commit to a capacity, and in the second stage firms simultaneously engage in Bertrand–Edgeworth competition. Under both efficient and proportional rationing, capacity commitment in the first stage permits both firms to avoid the Bertrand paradox in the second stage to earn positive profits. Under efficient rationing, capacity choice followed by Bertrand–Edgeworth competition leads, under fairly general conditions, to equilibrium prices that are identical to those that would arise in a Cournot (quantity setting) duopoly where firms' unit costs are the sum of capacity and production costs; see Kreps and Scheinkman (1983) and Deneckere and Kovenock (1996). Under proportional rationing, the Cournot outcome arises only if per unit capacity costs are sufficiently large. Otherwise, equilibria may arise in which capacities are asymmetric and non-degenerate mixed strategies are played at the pricing stage; see Davidson and Deneckere (1986).

Product differentiation

Bertrand competition with differentiated products is fundamentally different from Bertrand competition with homogenous products. With differentiated products, the demand for a firm's product is not generally discontinuous at p_L; a firm does not generally lose all of its demand by pricing slightly above p_L, nor does it steal all of rival firms' demands by pricing below p_L. In the classical model of differentiated-product Bertrand competition with downward sloping demands and costs that are non-decreasing in output, each firm's profit function, $\pi_i\left(p_i, p_{-i}\right)$, is assumed to be twice continuously differentiable, with $\partial\pi_i/\partial p_i\partial p_j > 0$ (strategic complements) and $\partial^2\pi_i/\partial p_i^2 < 0$.

With suitable assumptions on firms' demands and costs, a Bertrand equilibrium, $\left(p_i^*, p_{-i}^*\right)$, is simply the solution to the system of first-order conditions implied by each firm's profit-maximizing pricing decision:

$$\frac{\partial\pi_i\left(p_i^*, p_{-i}^*\right)}{\partial p_i} = 0 \text{ for all } i = 1, 2, \ldots n.$$

Alternatively, one may use the implicit function theorem and use firm i's first-order condition to obtain firm i's optimal price as a function of the prices charged by the other firms: $p_i = \rho_i\left(p_{-i}\right)$. The function ρ_i is called firm i's best-response (best-reply, reaction) function, and a Bertrand equilibrium in the case of differentiated products corresponds to the intersection of the firms' best-response functions. Total differentiation of firm i's first-order condition reveals that

$d\rho_i / dp_j = -\left(\partial\pi_i / \partial p_i \partial p_j\right) / \left(\partial^2 \pi_i / \partial p_i^2\right) > 0$; that is, strategic complementarities and the concavity of firm i's profits in p_i imply that firm i's best response function is upward sloping.

Notice that, at $\left(p_i^*, p_{-i}^*\right)$,

$$\frac{\partial\pi_i\left(p_i^*, p_{-i}^*\right)}{\partial p_i} = \left[p_i^* - C_i'\left(D_i\left(p_i^*, p_{-i}^*\right)\right)\right]\frac{\partial D_i\left(p_i^*, p_{-i}^*\right)}{\partial p_i} + D_i\left(p_i^*, p_{-i}^*\right) = 0.$$

Consequently, under mild regularity conditions firm i's equilibrium price exceeds its marginal cost. Furthermore, firms may charge different prices and earn positive profits in a differentiated product Bertrand equilibrium. These results may be extended to the case where $\pi_i\left(p_i, p_{-i}\right)$ is not differentiable by appealing to the more general notion of supermodularity (Vives, 1990; Milgrom and Roberts, 1990) rather than strategic complementarity (Bulow, Geanakoplos and Klemperer, 1985).

For the duopoly case with linear demands and constant unit costs, strategic complementarity ($\partial\pi_i / \partial p_i \partial p_i > 0$) arises naturally when the duopolists' products are substitutes in consumption ($\partial D_i / \partial p_j > 0$). In this case the firms' best-response functions are not only upward sloping (as is implied by strategic complementarity) but linear; consequently, there is a unique Bertrand equilibrium (see Cheng, 1985). Singh and Vives (1984) have shown that, in this linear duopoly case, even though each firm prices above its marginal cost in a differentiated-product Bertrand equilibrium, prices are lower under Bertrand competition than would arise in a differentiated-product Cournot (quantity setting) model. This result for linear demand and costs extends to markets with more than two firms when all firms' products are substitutes in consumption (Häckner, 2000).

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See also Cournot competition

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Index terms

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capacity
Cournot, A. A.
duopoly
Edgeworth cycles
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mixed-strategy equilibria
monopoly
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